Turbulent Time Scale for Turbulent-Flow Calculations

S. Zeierman* and M. Wolfshtein†
Technion—Israel Institute of Technology, Haifa, Israel

A turbulent time scale is proposed as the second turbulent variable in two-equation turbulence models. A governing equation for the time scale is derived and modeled. The modeling is based on a simple comparison of terms with the turbulent-energy equations. Solutions for one-dimensional cases agree with experimental data and facilitate estimates of the empirical constants.

Introduction

Background

TURBULENCE modeling has become widespread in the last decade and appears to be capable of supplying good answers to a large number of engineering problems. However, its reliability still remains weak. The major drawback is the lack of universality, which shows by the large variation of the degree of agreement with experiments for various cases and the need for adjustment of constants or introduction of empirical functions to take care of various effects. The practical answer to this problem has been the application of various empirical functions and constants to various situations. However, such a practice is not satisfactory because it implies that it is unsafe to calculate cases not measured previously. Thus, turbulence modeling becomes a complementary subject to experimental research and requires extensive supporting experiments. Under these circumstances, it is not surprising that improvements in turbulence models are still being sought.

Previous Work

The most vulnerable part of the subject is usually regarded as the scale equation. The first of these equations was Rotta's[1] length-scale equation, followed by other equations proposed for quantities regarded as functions of scale and turbulent energy. The most widely used of these quantities is the dissipation function (e.g., Harlow and Nakayama[2] or Jones and Launder[3]).

An exact governing equation for the dissipation may be derived from the Navier-Stokes equations. However, this equation is not very useful because its source terms (representing production and decay processes) are composed of complex correlations of fluctuating quantities, that are not easy to interpret or model (in fact, even the dissipation itself cannot be measured accurately). Therefore, the model equation for the dissipation is based on intuition and phenomenological arguments.

This situation is pointed out by Kline et al.,[4] who comment that "the weakest point of present one-point closure models is the dissipation equation," which needs some "tweaking of constants" in order to get good agreement with experimental data. They conclude that the dissipation equation is one of the points where special attention is required.

Under these conditions it appears advantageous to derive scale equations more amenable to theoretical treatment, and with measurable terms. It is true that some phenomenological arguments and simplifying assumptions may still be required. Yet such a formulation may become easier to model, and comparison of single terms of the equation with experiments is possible.

Objective

The present paper suggests that the turbulent integral time scale is the typical turbulence scale. The turbulent time scale is obtained by integration of the autocorrelation. Therefore, the time scale can be used only when the flow is stationary, or at least quasistationary. However, the derivation is valid for any spatial inhomogeneity. Moreover, the resulting equation is easier to model because the derivation is based on a small number of simplifying assumptions followed by an analysis. As the integral time scale is an easily measurable quantity, this practice appears to be more accessible to comparisons with experimental results. Therefore, this equation is a worthwhile candidate for numerical experimentation.

The present derivation is concerned with incompressible flows only.

Formulation

Definitions

In this work we use the usual splitting of all quantities into a mean part denoted by uppercase letters, and a fluctuating part denoted by lowercase letters. We use the following definitions for the autocorrelation and the Reynolds stresses:

$$R_{ij}(x,t') = \overline{u_i(x,t)u_j'(x,t+t')} \tag{1}$$

$$U_{ij} = R_{ij}(x,0) = \overline{u_i u_j} \tag{2}$$

where the unprimed quantities are at the time t, primed quantities are at time $t+t'$, and overbars denote time averages. Turbulent energy k and the turbulent time scale are defined by

$$k(x) = (1/2)R_{ii}(x,0) = (1/2)U_{ii} \tag{3}$$

$$T(x) = \frac{1}{4k}\int_{-\infty}^{\infty} R_{ii}(x,t')\,\mathrm{d}t' \tag{4}$$

For stationary processes, R_{ij} is symmetric with respect to t. Therefore, it is possible to write

$$T(x) = \frac{1}{2k}\int_{0}^{\infty} R_{ii}(x,t')\,\mathrm{d}t' \tag{5}$$

The time scale T represents the residence time of a particle within a typical large structure in a turbulent flowfield. This quantity is finite if R_{ij} vanishes for a long time as t^{-p} with $p>1$.

Received May 9, 1985; presented as Paper 85-1491 at the AIAA Seventh Computational Fluid Dynamics Conference, Cincinnati, OH, July 15-17, 1985; revision received Dec. 16, 1985.

*Student, Department of Aeronautical Engineering.
†Associate Professor, Department of Aeronautical Engineering.

Exact Time-Scale Equation

The governing equation for autocorrelation is obtained by multiplication of the fluctuating part of the Navier-Stokes equations at a given time t by the fluctuating velocity at time $t+t'$ and adding the result to the product of the equation at $t+t'$ by the velocity at t. Some algebra, as well as the assumption of quasistationarity, allows simplification of the final equations to

$$\frac{\partial R_{ii}}{\partial t}+U_j\frac{\partial R_{ii}}{\partial x_j}=-(R_{ij}+R_{ji})\frac{\partial U_i}{\partial x_j}$$
$$-\frac{1}{2}\frac{\partial}{\partial x_j}(S_{ij,i}+S_{i,ij})-\frac{1}{\rho}\left(\frac{\partial K_{p,j}}{\partial x_j}+\frac{\partial K_{j,p}}{\partial x_j}\right)$$
$$+\nu\frac{\partial^2 R_{ii}}{\partial x_j^2}-2\nu\overline{\frac{\partial u_i}{\partial x_j}\frac{\partial u_i'}{\partial x_j}} \tag{6}$$

where S and K are triple- and pressure-velocity autocorrelations, respectively. Our two-equation model is based on the turbulent energy and time scale. Both quantities can be derived from Eq. (6). The energy equation, obtained by making $t'=0$, is

$$\frac{Dk}{Dt}=\frac{\partial}{\partial x_j}\left(-\overline{u_i^2u_j}-\frac{1}{\rho}\overline{pu_j}+\nu\frac{\partial k}{\partial x_j}\right)+G-\epsilon \tag{7}$$

where

$$G=-U_{ij}\frac{\partial U_i}{\partial x_j} \tag{8}$$

$$\epsilon=\nu\overline{\left(\frac{\partial u_i}{\partial x_j}\right)^2} \tag{9}$$

This equation represents a balance between convection, diffusion, production G, and dissipation ϵ, of turbulence energy.

The time-scale equation, obtained by integration of Eq. (6) on dt', is

$$\frac{D(kT)}{Dt}= \qquad \text{I}$$
$$-\frac{1}{2}\frac{\partial}{\partial x_j}\int_0^\infty\left(S_{ij,i}+S_{i,ij}+\frac{1}{\rho}K_{p,j}+\frac{1}{\rho}K_{j,p}\right)dt' \qquad \text{II}$$
$$+\nu\frac{\partial^2(kT)}{\partial x_j^2} \qquad \text{III}$$
$$-\frac{1}{2}\int_0^\infty(R_{ij}+R_{ji})\frac{\partial U_i}{\partial x_j}dt' \qquad \text{IV}$$
$$-\nu\int_0^\infty\overline{\frac{\partial u_i}{\partial x_j}\frac{\partial u_i'}{\partial x_j}}dt' \qquad \text{V} \tag{10}$$

In this equation, term I represents convection of kT, II represents turbulent diffusion of kT, III represents viscous diffusion, IV represents production, and V represents viscous decay.

Modeling

In this section we shall discuss the modeling of Eqs. (7) and (10) for the turbulent energy and the time scale. The present modeling is restricted to high turbulence levels, and therefore, is not applicable to the viscous sublayer near solid walls or low-level turbulence as found immediately after transition from laminar to turbulent flow.

Modeling of the Energy Equation

Equation (7) contains three terms that require modeling, namely, diffusion, production G, and dissipation ϵ. (We will discuss diffusion later.) The production term depends on the Reynolds stresses. In all two-equation models an eddy-viscosity hypothesis is used to calculate the Reynolds stresses. All existing models follow Prandtl's suggestion that the turbulent viscosity is proportional to the product of the square root of the energy and a length scale. The length scale of the large turbulent structures is equal to the relative velocity of the particles in the structure multiplied by the time scale:

$$L\alpha k^{1/2}T \tag{11}$$

This definition allows us to calculate the turbulent viscosity as follows:

$$\nu_t=C_\mu kT \tag{12}$$

where C_μ is an empirical constant. The next term to be considered is the turbulent-energy dissipation. In flows with high turbulence levels the dissipation is usually assumed to be isotropic. By substitution of Eq. (11) into the inviscid estimate for dissipation, which is proportional to $k^{3/2}/L$, we get

$$\epsilon=C_d k/T \tag{13}$$

where C_d is another empirical constant.

Modeling of the Time-Scale Equation

In the time-scale equation (10) we have three new terms that require modeling: namely, the diffusion term, II; the production term, IV; and the viscous decay term, V. We will show that the two latter terms are related to the production and dissipation terms in the turbulent-energy equation.

We shall start with the production term. For a stationary process, we can write $R_{ij}=R_{ji}$. Thus, the problem is how to integrate the components of the autocorrelation tensor. However, the components of this tensor are not defined in the context of the present model. We need the trace only, as it appears in the definition of the time-scale equation (4). This difficulty may be overcome by using a model to relate the autocorrelation components to the Reynolds stress. In general linear relation we need a fourth-order tensor to perform this transformation. However, if we are willing to accept a relationship based on scalars we can get:

$$R_{ij}(x,t')=\alpha(t')U_{ij}(x)+\beta(t')k(x)\delta_{ij} \tag{14}$$

The autocorrelation tensor calculated from Eq. (14) is symmetrical, which is true for stationary flows. The dimensionless quantities α and β are symmetric functions of separation in time t' only. Thus,

$$\alpha(t')=\alpha(-t')$$

with a similar relation for β. Once again, this property is true for stationary flows only.

Substitution of Eq. (14) into the production term IV gives

$$\text{IV}=G\int_0^\infty\alpha(t')dt' \tag{15}$$

where G is defined in Eq. (8). The integral of the dimensionless quantity α has the dimension of time. Since it represents large eddy processes, this integral is assumed to be proportional to the time scale, and the final production term, IV, is

$$\text{IV}=C_g TG \tag{16}$$

where C_g is an empirical constant.

The next term to be modeled is the decay. Examination of the decay term reveals that the integrand approaches the turbulent-energy dissipation when t' approaches zero. Therefore, the decay term is assumed to be equal to the dissipation multiplied by a quantity that has the dimension of time. We shall refer to it as the "time scale of the decay term." To close the system, we will relate this time scale to the turbulent time scale T defined in Eq. (4). Within the framework of a two-dimensional model the ratio of these two time scales should be either a constant or a function of the turbulence Reynolds number. It is usually assumed that when the turbulence level is high and an inertial subrange exists, the turbulent-energy dissipation does not depend on the viscosity. We suggest an extension of this assumption to the present situation by assuming that the time-scale ratio of the energy carrying eddies to the decay time scale is constant. Therefore, the model for the decay term V is

$$V = C_e T \epsilon \tag{17}$$

where C_e is an empirical constant.

Modeling of the Diffusion Terms

Finally, we consider the turbulent diffusion terms in the turbulent-energy and time-scale equations. These terms are flux gradients. It has become customary to model fluxes as gradients of the transferred quantities, and we propose to follow this practice here as well. Therefore, the model for the diffusion flux of k is

$$\frac{\partial}{\partial x_j}\left(\frac{\nu_t}{\sigma_k}\frac{\partial k}{\partial x_j}\right) \tag{18}$$

where σ_k is an empirical constant representing a turbulent Prandtl number. The model for the diffusion flux of kT is

$$\frac{\partial}{\partial x_j}\left(\frac{\nu_t}{\sigma_T}\frac{\partial (kT)}{\partial x_j}\right) \tag{19}$$

where σ_T is yet another empirical constant.

Final Model

With all of these substitutions, the modeled turbulent-energy equation is

$$\frac{\mathrm{D}k}{\mathrm{D}t} = \frac{\partial}{\partial x_j}\left[\left(\nu + \frac{\nu_t}{\sigma_k}\right)\frac{\partial k}{\partial x_j}\right] + \nu_t \frac{1}{2}\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right)^2 - C_d \frac{k}{T} \tag{20}$$

The modeled time-scale equation is

$$\frac{\mathrm{D}(kT)}{\mathrm{D}t} = \frac{\partial}{\partial x_j}\left[\left(\nu + \frac{\nu_t}{\sigma_T}\right)\frac{\partial (kT)}{\partial x_j}\right] + C_g T \nu_t \frac{1}{2}\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right)^2 - C_e C_d k \tag{21}$$

Equations (20) and (21) form the complete model. The total number of constants in the model appears to be six; namely, C_d, C_μ, C_g, C_e, σ_k, and σ_T. However, the viscosity and decay constants C_μ and C_d always appear together with the time scale T. Therefore one of them may be arbitrarily fixed without any loss of generality (if it is acceptable to replace the integral scale by a quantity proportional to it). Thus we must estimate two constants for the turbulent-energy equation and three constants for the time-scale equation. Note that the molecular viscosity in the diffusion terms of Eqs. (20) and (21) is negligible for high levels of turbulence discussed herein. However, we left the viscosity in the formulation because it is an exact consequence of the derivation.

Representative Solutions

In this section we shall obtain analytical solutions of Eqs. (20) and (21) for some simple problems, where such solutions are possible. These solutions will be compared with experimental data, and some of the empirical constants will be deduced. All of the cases are simple cases governed by ordinary differential equations and, apart from the first one, are stationary and incompressible.

Decay of Isotropic Turbulence

This is a nonstationary flow. However, the temporal variations are rather slow and the problem may be regarded as quasistationary. The turbulence is homogeneous and without mean-velocity gradients. Therefore, the diffusion and production terms vanish. The k and kT equations in this case are

$$\frac{\mathrm{d}k}{\mathrm{d}t} = -C_d k/T \tag{22}$$

$$\frac{\mathrm{d}(kT)}{\mathrm{d}t} = -C_e C_d k \tag{23}$$

and the solution is

$$T = T_0 + C_d(1-C_e)t \tag{24}$$

$$\frac{k}{k_0} = \left(\frac{T_0}{C_d}\right)^{1/(1-C_e)}\left[\frac{T_0}{C_d} + (1-C_e)t\right]^{-1/(1-C_e)} \tag{25}$$

In the literature turbulent-energy decay is given by formulas of the form $k \alpha t^{-p}$. By comparison with the present solution we get

$$C_e = (p-1)/p \tag{26}$$

The data of Comte-Bellot and Corrsin[5] suggests $p = 1.29$. The corresponding value of C_e is 0.225.

Logarithmic Boundary Layers

In this case, the turbulent shear stress and turbulent energy are uniform. Integration of the momentum equation yields

$$\frac{\mathrm{d}U}{\mathrm{d}y} = \frac{(v^*)^2}{\nu_t} \tag{27}$$

where $(v^*)^2 = \tau_w/\rho$. The governing equations are

$$\frac{\mathrm{d}}{\mathrm{d}y}\left(\frac{C_\mu kT}{\sigma_k}\frac{\mathrm{d}k}{\mathrm{d}y}\right) + C_\mu kT\left(\frac{\mathrm{d}U}{\mathrm{d}y}\right)^2 - C_d\frac{k}{T} = 0 \tag{28}$$

$$\frac{\mathrm{d}}{\mathrm{d}y}\left(\frac{C_\mu kT}{\sigma_T}\frac{\mathrm{d}(kT)}{\mathrm{d}y}\right) + C_g C_\mu kT^2\left(\frac{\mathrm{d}U}{\mathrm{d}y}\right)^2 - C_e C_d k = 0 \tag{29}$$

The solution is

$$k = (v^*)^2/(C_\mu C_d)^{1/2} \tag{30}$$

$$T = \frac{y}{v^*}\left[\frac{\sigma_T(C_e - C_g)C_d^{3/2}}{C_\mu^{1/2}}\right]^{1/2} \tag{31}$$

This flow is very well documented in the experimental literature. The velocity gradient, turbulent viscosity, and turbulent energy are given by

$$\frac{\mathrm{d}U}{\mathrm{d}y} = \frac{v^*}{\kappa y} \tag{32}$$

$$\nu_t = \kappa v^* y \tag{33}$$

$$(v^*)^2/k = 0.3 \tag{34}$$

with $\kappa = 0.41$. By comparison of these formulas with the theoretical solutions, the following relations are obtained:

$$C_d C_\mu = 0.09 \tag{35}$$

$$C_g = C_e - \frac{\kappa^2 [k/(v^*)^2]}{\sigma_T} = C_e - \frac{0.56}{\sigma_T} \tag{36}$$

Flow in the Center of a Duct

In this case we consider a fully developed flow in a two-dimensional duct. The shear stress is linear with respect to the distance from the duct axis y. Near the wall the shear stress is nearly constant, and the solution is practically identical to the logarithmic boundary layer. However, near the center of the duct the shear stress and the production vanish. The velocity and time scale reach a maximum on the axis, while the turbulent energy has a minimum there. The polynomial expansions of all three quantities should have even powers only, due to the symmetry. Essentially this is a region of balance between diffusion and decay of all turbulent quantities. It is more convenient to analyze this flow using a new variable $n = kT$. Using this definition, the governing equations are

$$\frac{d}{dy}\left(\frac{C_\mu}{\sigma_k} n \frac{dk}{dy}\right) - C_d \frac{k^2}{n} = 0 \tag{37}$$

$$\frac{d}{dy}\left(\frac{C_\mu}{\sigma_T} n \frac{dn}{dy}\right) - C_e C_d k = 0 \tag{38}$$

The transformation $dy = n dz$ simplifies the equations to

$$\frac{d^2 k}{dz^2} - A k^2 = 0 \tag{39}$$

$$\frac{d^2 n}{dz^2} - B k n = 0 \tag{40}$$

where

$$A = (C_d / C_\mu) \sigma_k \tag{41}$$

$$B = (C_d / C_\mu) C_e \sigma_T \tag{42}$$

The solution is

$$\frac{k}{k_0} = 1 + \frac{A}{2} \frac{k_0}{n_0^2} y^2 + \mathcal{O}(y^4) \tag{43}$$

$$\frac{n}{n_0} = 1 + \frac{B}{2} \frac{k_0}{n_0^2} y^2 + \mathcal{O}(y^4) \tag{44}$$

Using these values the velocity profile is

$$\frac{U}{U_0} = 1 - \frac{1}{2C_\mu} \frac{p'}{n_0 U_0} y^2 + \mathcal{O}(y^4) \tag{45}$$

where p' is the longitudinal pressure gradient.

Wolfshtein[6] deduced the following formulas from the experimental data of Clark[7] and Laufer[8]:

$$\frac{k}{k_0} = 1 + 6.67 \left(\frac{y}{h}\right)^2 \tag{46}$$

$$\frac{U}{U_0} = 1 - 0.242 \left(\frac{y}{h}\right)^2 \tag{47}$$

where $2h$ is the duct width

$$(v^*)^2 / U_0 k_0 = 0.048 \tag{48}$$

Comparison of these formulas with the theoretical solution shows that the model yields the correct distributions. By elimination of n_0 we can get the following relation between the constants:

$$\sigma_k C_\mu C_d = 0.13 \tag{49}$$

and if we utilize the value of $C_\mu C_d = 0.09$ already recommended we get $\sigma_k = 1.46$.

Flow with a High-Pressure Gradient

When the pressure gradient p' is very large it often happens that $p'y \gg \tau_w$, where y is the distance from the wall and τ_w is the skin friction. In this case, shear stress is nearly linear in the distance from the wall y and

$$\frac{dU}{dy} = \frac{p'y}{\nu_t} \tag{50}$$

It will be convenient to use the variable $n = kT$ and obtain the governing equations in the following form:

$$\frac{d}{dy}\left(\frac{C_\mu}{\sigma_k} n \frac{dk}{dy}\right) + \frac{p'^2 y^2}{C_\mu n} - C_d \frac{k^2}{n} = 0 \tag{51}$$

$$\frac{d}{dy}\left(\frac{C_\mu}{\sigma_T} n \frac{dn}{dy}\right) + C_g \frac{p'^2 y^2}{C_\mu k} - C_e C_d k = 0 \tag{52}$$

The solution is

$$k = Ay \tag{53}$$

$$n = By^{1.5} \tag{54}$$

with

$$A = p' \left[\frac{(2\sigma_k - \sigma_T C_g)}{C_\mu C_d (2\sigma_k - \sigma_T C_e)}\right]^{1/2} \tag{55}$$

$$B^2 = \frac{2\sigma_k}{3C_\mu} p' \left(\frac{C_d}{C_\mu}\right)^{1/2} \frac{\rho - 1}{\rho}, \quad \rho = \left[\left(\frac{2\sigma_k - \sigma_T C_g}{2\sigma_k - \sigma_T C_e}\right)\right]^{1/2} \tag{56}$$

Substitution in the momentum equation gives

$$U = 2p' \sqrt{y} / C_\mu B \tag{57}$$

This should be compared with Townsend's[9] experimental data

$$U = 2\sqrt{p'y} / K_0 \tag{58}$$

with $K_0 = 0.48$. The comparison yields

$$K_0 = \left(\frac{2}{3}\sigma_k\right)^{1/2} (C_\mu C_d)^{1/4} \left(\rho - \frac{1}{\rho}\right)^{1/2} \tag{59}$$

Substitution of the other constants yields $\sigma_T = 10.8$.

Uniform Strain Flow

Uniform strain flows are very attractive to theoreticians because the solution of the governing equations of the turbulence properties is much simpler in such flows. The simplest uniform strain flow is the flow with a single velocity gradient dU/dy. This flow was studied experimentally by some researchers. The analysis is much simpler for the fully developed case where the only two processes are production and decay. Unfortunately, the experiments did not show conclusively whether this situation is possible. Therefore we shall add the convection terms. Under these conditions the equations for the turbulence energy and the time scale are

$$\frac{\mathrm{d}k}{\mathrm{d}x}=\frac{1}{U}\left(\frac{\mathrm{d}U}{\mathrm{d}y}\right)^2 C_\mu kT-\frac{1}{U}C_d\frac{k}{T} \tag{60}$$

$$\frac{\mathrm{d}T}{\mathrm{d}x}=-\frac{1}{U}\left(\frac{\mathrm{d}U}{\mathrm{d}y}\right)^2(1-C_g)C_\mu T^2+\frac{1}{U}C_d(1-C_e) \tag{61}$$

the solution of T is

$$T=A\cdot tgh(Bx+C) \tag{62a}$$

with

$$A=\left(1\Big/\frac{\mathrm{d}U}{\mathrm{d}y}\right)\left[\frac{C_d}{C_\mu}\frac{(1-C_e)}{(1-C_g)}\right]^{1/2} \tag{62b}$$

$$B=\frac{1}{U}\frac{\mathrm{d}U}{\mathrm{d}y}[C_dC_\mu(1-C_g)(1-C_e)]^{1/2} \tag{62c}$$

$$C=tgh^{-1}(T_0/A) \tag{62d}$$

When x is very small T may be shown to be proportional to x, as reported by Mulhearn and Luxton.[10] When x is very large the asymptotic form of the solution is

$$T=\left(1\Big/\frac{\mathrm{d}U}{\mathrm{d}y}\right)\left[\frac{C_d}{C_\mu}\frac{(1-C_e)}{(1-C_g)}\right]^{1/2} \tag{63}$$

The experiment appears not to have reached the fully developed stage. Therefore no comparison with experiments is possible. After substitution of T into the energy equation, the solution of k is as follows:

$$\frac{k}{k_0}=\left(\frac{\cosh(Bx+C)}{\cosh(C)}\right)^{1/(1-C_g)}\left(\frac{\sinh(Bx+C)}{\sinh(C)}\right)^{-[1/(1-C_e)]} \tag{64}$$

The asymptotic form of this solution for very large x is

$$k\sim[\sinh(Bx+C)]^{(C_g-C_e)/[(1-C_g)(1-C_e)]} \tag{65}$$

and as $C_e>C_g$, k tends to zero for very large x.

Discussion and Conclusions

This paper presents a proposal for a new scale equation for complex turbulent flows. Such an equation is desired in view of the lack of universality and confidence in existing scale equations. Even the dissipation equation, which is the most popular choice of a scale equation at the present time, does not give a satisfactory solution. The exact solutions presented indicate the capabilities of the model for flows with a high turbulence level. The model handles duct flows near the duct axis and near the wall. It can cope with the logarithmic boundary layer, which is of typical zero-pressure gradient flows, and high-pressure gradients as well. Simple homogeneous flows were also calculated for cases with and without production of turbulence. Of course, this is not sufficient to validate the model and numerical calculation of more complex flows will have to be undertaken. However, the following values for the constants, which were obtained by comparison of the theoretical solutions with the experimental data, may serve for more complex cases until better values become available: $C_\mu=0.09$, $C_d=1.0$, $C_g=0.173$, $C_e=0.225$, $\sigma_k=1.46$, $\sigma_T=10.8$.

As previously mentioned, C_μ and C_d should be selected such that the value of the computed and measured time scales is identical. However, since experimental data on this quantity are not readily available, we chose not to satisfy this requirement at the present time. Therefore, it is possible to specify one of these two constants without loss of generality. We chose to specify C_d arbitrarily as 1.

The autocorrelation equation is relatively simple, and the number of terms which require modeling is smaller than in other scale equations. Moreover, the modeling of its terms is based on an attempt to utilize physical understanding of the situation. Most terms of this equation are related to corresponding terms in the turbulent-energy equation. A definite advantage of the proposed model is that the autocorrelation (or its Fourier transform, the power spectrum) and the time scale are easily measurable quantities and are more readily available from experiments than corresponding quantities in other models. Therefore, wider comparisons with experimental data are possible. Moreover, the fact that the definition of the autocorrelation is entirely independent of spatial directions suggests that the time-scale equation may be easier to model in the viscous sublayer adjacent to solid walls.

It is of interest to note that the time-scale equation may be regarded as a turbulent-viscosity equation. Nee and Kovasznay[11] have proposed an equation for this quantity. They did not have an exact equation for the turbulent viscosity and their modeling was based on dimensional and qualitative arguments only. Moreover, they used only this equation, without the turbulent-energy equations, although they did recognize the need for another turbulent quantity, which they satisfied by using an empirical length-scale distribution. As a result, the models they used for the production, decay, and turbulent diffusion terms are different from those presented herein. The present derivation is more appealing to the present authors because they received some guidance from the exact equation, the assumptions required are less bold, and the modeling techniques are in line with contemporary models of these terms.

In conclusion, we find the time-scale equation to be an attractive option and consider it a worthwhile candidate for numerical experimentation.

References

[1]Rotta, J., "Statistische Theorie Nichthomogener Turbulenz," *Zeitschrift fuer Physik*, Vol. 129, 1951, pp. 547-572; also, Vol. 131, 1951, pp. 51-77.

[2]Harlow, F. H. and Nakayama, P. I., "Turbulent Transport Equations," *The Physics of Fluids*, Vol. 10, No. 11, 1967, p. 2323.

[3]Jones, W. P. and Launder, B. E., "The Prediction of Laminarization with a Two-Equation Model of Turbulence," *International Journal of Heat and Mass Transfer*, Vol. 15, 1972, p. 301.

[4]Kline, S. J., Cantwell, B. J., and Lilley, G. M., *The 1980-81 AFOSR-HTTM-Stanford Conference on Complex Turbulent Flows*, Vol. II, Stanford University, Stanford, CA, 1982, p. 980.

[5]Comte-Bellot, J. and Corrsin, S., "The Use of a Contraction to Improve the Isotropy of Grid Generated Turbulence," *Journal of Fluid Mechanics*, Vol. 25, No. 4, 1966, pp. 657-682.

[6]Wolfshtein, M., "On the Length-Scale-of-Turbulence Equation," *Israel Journal of Technology*, Vol. 8, No. 1-2, 1970, pp. 87-99.

[7]Clark, J. A., "A Study of Incompressible Turbulent Boundary Layers in Channel Flows," *Transactions of the American Society of Mechanical Engineers*, Paper 68-FE-26, 1968.

[8]Laufer, J., "Some Recent Measurements in a Two-Dimensional Turbulent Channel," *Journal of the Aeronautical Sciences*, Vol. 17, 1950, pp. 277-287.

[9]Townsend, A. A., "Equilibrium Boundary Layers and Wall Turbulence," *Journal of Fluid Mechanics*, Vol. 11, 1961, pp. 97-120.

[10]Mulhearn, P. J. and Luxton, R. E., "The Development of Turbulence Structure in a Uniform Shear Flow," *Journal of Fluid Mechanics*, Vol. 68, No. 3, 1975, pp. 577-590.

[11]Nee, V. W. and Kovasznay, L.S.G., "Simple Phenomenological Theory of Turbulent Shear Flows," *The Physics of Fluids*, Vol. 12, No. 3, 1969, p. 473.